Precalc Warm Up # 9-2

Graph without using a grapher, and label all asymptotes, holes and intercepts.

1. $f(x)=\dfrac{2x^2+6x}{x^2-9}$

2. Divide $2x^2 + x - 3$ by $x + 2$

and graph $f(x)=\dfrac{2x^2+x-3}{x+2}$

3. Which are the improper fractions?

$$\frac{2}{6} \quad , \quad \frac{10}{8} \quad , \quad \frac{x+4}{x^2+7} \quad , \quad \frac{x^2-5x+4}{x+7}$$

Graph without using a grapher, and label all asymptotes, holes and intercepts.

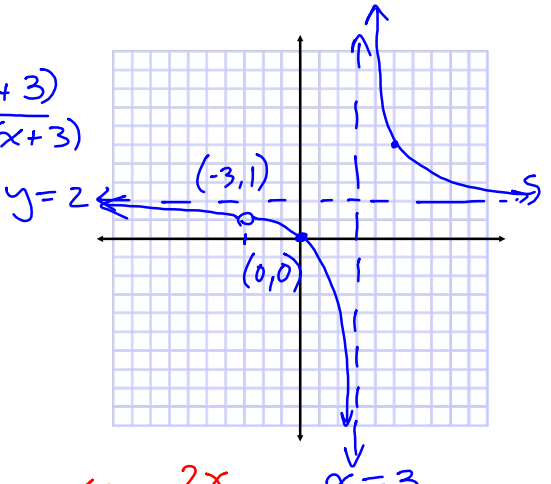
1. $f(x)=\dfrac{2x^2+6x}{x^2-9} = \dfrac{2x(x+3)}{(x-3)(x+3)}$

Vert: $x = 3$

horiz: $y = 2$

hole: $x = -3$

x	y
0	0
5	$\frac{50+30}{25-9} = 5$



$h(x)=\dfrac{2x}{x-3}$

$h(-3)=\dfrac{2(-3)}{-3-3} = \dfrac{-6}{-6} = 1$

2. Divide $2x^2 + x - 3$ by $x + 2$

$0 = 2x^2 + x - 3$

$0 = (2x + 3)(x - 1)$

and graph $f(x) = \dfrac{2x^2 + x - 3}{x + 2}$

$18 - 3 - 3$

-1

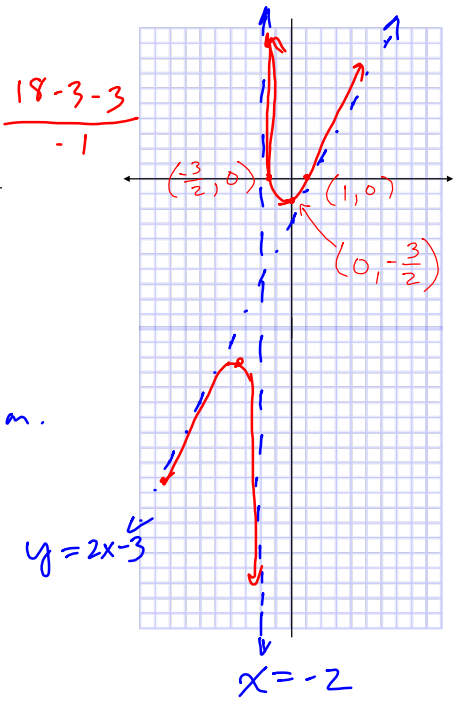
$-2 \mid 2 \quad 1 \quad -3$

$\quad\quad -4 \quad 6$

$\quad 2 \quad -3 \quad 3$ rem.

$y = 2x - 3$

x	y
0	$-3/2$
1	0
$-\frac{3}{2}$	0
-3	-12



3. Which are the improper fractions?

$$\frac{2}{6}, \quad \frac{10}{8}, \quad \frac{x+4}{x^2+7}, \quad \frac{x^2-5x+4}{x+7}$$

Similar idea

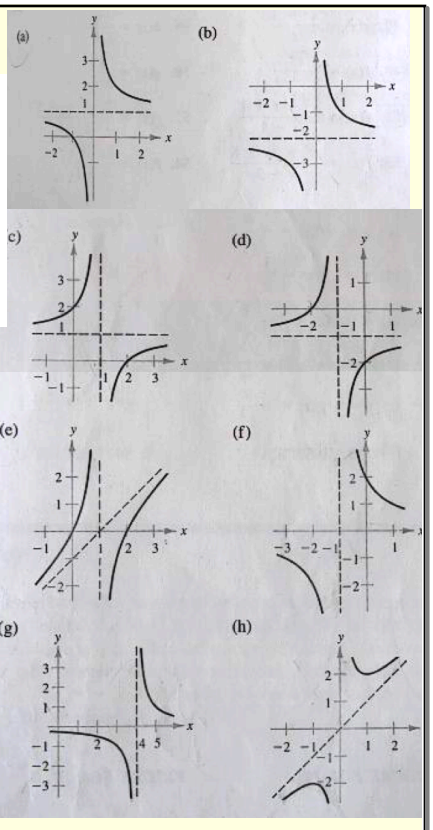
when degree top > degree bottom

it's called an improper rational expression.

November 01, 2016

HW Questions:
Matching

1. $f(x) = \frac{2}{x+1}$
2. $f(x) = \frac{1}{x-4}$
3. $f(x) = \frac{x+1}{x}$
4. $f(x) = \frac{1-2x}{x}$
5. $f(x) = \frac{x-2}{x-1}$
6. $f(x) = -\frac{x+2}{x+1}$
7. $f(x) = \frac{x^2+1}{x}$
8. $f(x) = \frac{x^2-2x}{x-1}$



In Exercises 9–18, find the domain of the function and identify any horizontal, vertical, or slant asymptotes.

11. $f(x) = \frac{2+x}{2-x}$

15. $f(x) = \frac{3x^2+1}{x^2+9}$

In Exercises 21–44, sketch the graph of the rational function. As sketching aids, check for intercepts, symmetry, vertical asymptotes, and horizontal asymptotes.

23. $h(x) = \dfrac{-1}{x+2}$

25. $f(x) = \dfrac{x+1}{x+2}$

29. $f(t) = \dfrac{3t+1}{t}$

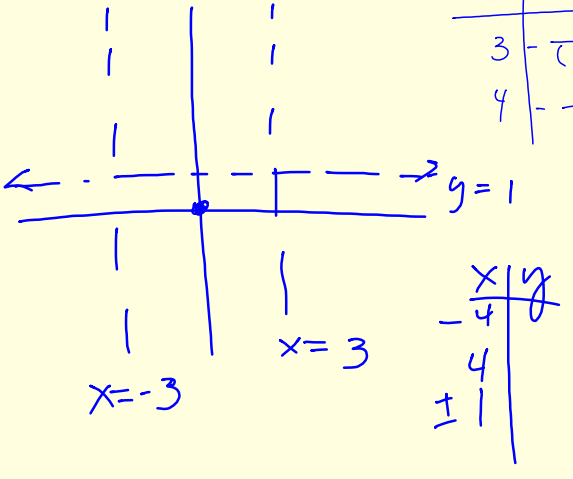
33. $C(x) = \dfrac{5+2x}{1+x}$

37. $h(x) = \dfrac{x^2}{x^2-9}$

37) horiz @ $y=1$

Vert: $x=\pm 3$

$(0,0)$



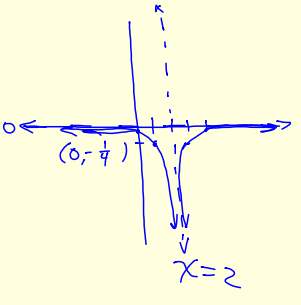
x	y
4	
4	
±1	

41. $f(x) = -\dfrac{1}{(x-2)^2}$

Vert: $x=2$

Horiz: $y=0$

x	y
0	$-\frac{1}{4}$
1	$-\frac{1}{1} = -1$
3	$-\frac{1}{(1)^2} = -1$
4	$-\frac{1}{2^2} = -\frac{1}{4}$



45. $f(x) = \dfrac{2x^2 + 1}{x}$

49. $f(x) = \dfrac{x^3}{x^2 - 1}$

51. $f(x) = \dfrac{x^2 - x + 1}{x - 1}$

$x^2 - 1 \overline{)\, x^3 + 0x^2 + 0x + 0}$ $\quad x$

$-(x^3 \quad -x)$

$x + 0$ rem.

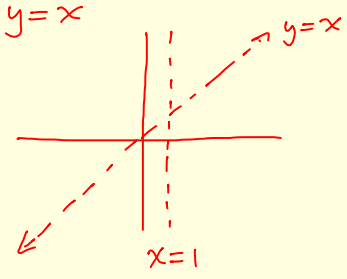
vert

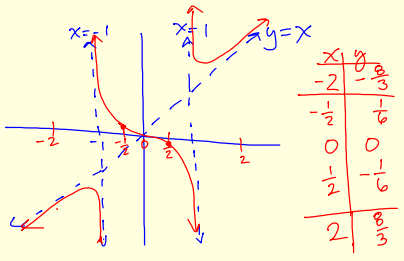
$x = 1$ and $x = -1$ Slant: $y = x$

Vert @ $x = 1$

1 | 1 | −1 | 0

1 | 0 | 1 rem.





55. The game commission introduces 50 deer into newly acquired state game lands. The population of the herd is given by

$$N = \frac{10(5 + 3t)}{1 + 0.04t}, \qquad 0 \le t$$

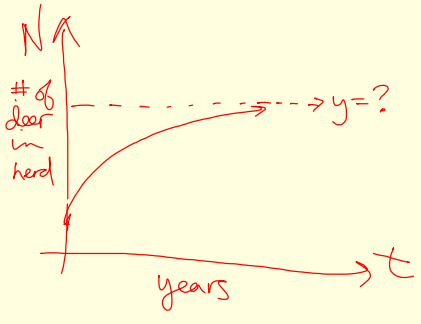
where t is time in years.

(a) Find the population when t is 5, 10, and 25.

(b) What is the limiting size of the herd as time increases?

asking if there is a horizontal asymptote.

$y = \dfrac{30}{0.04} = 750$



59. A right triangle is formed in the first quadrant by the x-axis, the y-axis, and a line segment through the point $(2, 3)$. (See figure.)

(a) Show that an equation of the line segment is

$$y = \frac{3(x - a)}{2 - a}, \qquad 0 \le x \le a.$$

use pt-slope form with $(a, 0)$ & m

$$m = \frac{3 - 0}{2 - a}$$

$$y - 0 = \frac{3}{2-a}(x - a)$$

$$y = \frac{3(x-a)}{2-a}$$

(b) Show that the area of the triangle is

$$A = \frac{-3a^2}{2(2 - a)}.$$

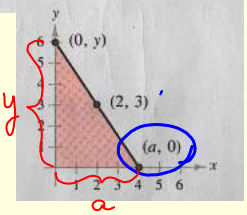
(c) Sketch the graph of the area function of part (b), and from the graph estimate the value of a that yields a minimum area. Use grapher → 2nd Calculate min.

$$-2\left(\frac{y-3}{-2}\right) = \left(\frac{3-0}{2-a}\right)(-2)$$

$$y - 3 = \frac{-6}{2-a}$$

$$y = -\frac{6}{2-a} + \frac{3(2-a)}{2-a}$$

$$y = \frac{3a}{2-a}$$



$$A = \frac{1}{2}ay$$

$$A = \frac{1}{2}a\left(\frac{-3a}{2-a}\right)$$

Add: $\dfrac{2}{x-3} + \dfrac{-1}{x+2}$ LCD $= (x-3)(x+2)$

$$\frac{2}{(x-3)} \cdot \frac{(x+2)}{(x+2)} + \frac{-1}{(x+2)} \cdot \frac{(x-3)}{(x-3)}$$

$$\frac{2x+4-x+3}{(x+2)(x-3)}$$

$$\boxed{\frac{x+7}{x^2-x-6}}$$

Today we will reverse the process.

We will take $\dfrac{x+7}{x^2-x-6}$

and rewrite it as $\dfrac{2}{x-3}+\dfrac{-1}{x+2}$

This process is called
PARTIAL FRACTION DECOMPOSITION,
and you will have 5 different types of problems.

Type 1: Distinct linear factors of the denominator
Write the partial fraction decomposition for

$$\frac{x+7}{x^2-x-6} = \frac{A(x+2)}{(x-3)(x+2)} + \frac{B}{(x+2)}\cdot\frac{(x-3)}{(x-3)}$$

$(x-3)(x+2)$

$$\frac{x+7}{(x-3)(x+2)} = \frac{A(x+2)+B(x-3)}{(x-3)(x+2)}$$

Basic Eq. ★ $x+7 = A(x+2)+B(x-3)$

Let $x=3$ $\quad 3+7 = A(3+2)+B(3-3)$

$10 = 5A \rightarrow \boxed{A=2}$

Let $x=-2 \rightarrow -2+7 = 0 + B(-2-3)$

$5 = B(-5) \rightarrow \boxed{B=-1}$

$$\frac{x+7}{x^2-x-6} = \frac{2}{x-3} + \frac{-1}{x+2}$$

Type 2: Repeated linear factors
Write the partial fraction decomposition for

$$\frac{5x^2+20x+6}{x^3+2x^2+x} = \frac{A(x+1)^2}{x(x+1)^2} \quad \frac{B}{x+1} + \frac{C}{(x+1)^2}$$

$x(x^2+2x+1)$
$x(x+1)^2$

$$5x^2+20x+6 = A(x+1)^2 + Bx(x+1) + Cx$$

Let $x=0$ $\longrightarrow$ $\boxed{6=A}$

Let $x=-1$ $\quad 5-20+6 = C(-1)$

$-9=-C \rightarrow \boxed{C=9}$

Let $x=1 \rightarrow 5+20+6 = 6(4)+B(1)(2)+9(1)$

$31 = 24+2B+9$

$-2 = 2B \rightarrow \boxed{B=-1}$

$$\boxed{\frac{5x^2+20x+6}{x^3+2x^2+x} = \frac{6}{x} + \frac{-1}{x+1} + \frac{9}{(x+1)^2}}$$

Type 3: Distinct linear and Quadratic Factors
Write the partial fraction decomposition for

$$\frac{3x^2+4x+4}{x^3+4x} = \frac{A}{x} + \frac{Bx+C}{x^2+4}$$

$x(x^2+4)$

$$3x^2+4x+4 = A(x^2+4) + (Bx+C)x$$

Let $x=0$ $\longrightarrow$ $4=4A \rightarrow \boxed{A=1}$

Let $x=1 \rightarrow 3+4+4 = 1(1+4) + B+C$

$\boxed{6 = B+C}$

Let $x=-1 \rightarrow 3-4+4 = 1(5) + (-B+C)(-1)$

$\boxed{-2 = B-C}$

$4 = 2B \rightarrow \boxed{B=2}$ $\boxed{C=4}$

$$\frac{3x^2+4x+4}{x^3+4x} = \frac{1}{x} + \frac{2x+4}{x^2+4}$$

Type 4: Repeated Quadratic Factors

Write the partial fraction decomposition for

$$\frac{8x^3+13x}{(x^2+2)^2} = \frac{Ax+B}{(x^2+2)} + \frac{Cx+D}{(x^2+2)^2}$$

$$8x^3 + 13x = (Ax+B)(x^2+2) + Cx + D$$

Notice: $8x^3 = Ax^3$; so $\boxed{A=8}$.

Also: No constant on the left, so $\boxed{D=0}$

Can you see what B is?

Type 5: Improper rational expressions, divide first!

Write the partial fraction decomposition for

$$\frac{2x^3+x^2-7x+7}{x^2+2x-3}$$

$$= 2x - 3 + \frac{5x-2}{x^2+2x-3}$$

$$\begin{array}{r} 2x-3 \\ x^2+2x-3\overline{)\,2x^3+x^2-7x+7} \\ -(2x^3+4x^2-6x) \\ \hline -3x^2-x+7 \\ -(-3x^2-6x+9) \\ \hline 5x-2 \end{array}$$

Now decompose the fraction part:

$$\left(\frac{5x-2}{(x+3)(x-1)}\right)^{LCD} = \left(\frac{A}{x+3} + \frac{B}{x-1}\right)^{LCD}$$

Basic Equation:

$$5x-2 = A(x-1) + B(x+3)$$

Now choose convenient values for x so you can find A & B.

$$\frac{2x^3+x^2-7x+7}{x^2+2x-3} = 2x - 3 + \frac{17}{4(x+3)} + \frac{3}{4(x-1)}$$

HW: PC Book

p. 246 (at the top of the page),

#3 - 35 □, skip 23

HW Quiz
Week 8
tomorrow.